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Relevance of the Application of the Theory of Fuzzy Sets in the Calculation of the Strategic Security of a Complex Technical System

Abstract: The relevance of the topic is related to the problem of protection and security of technical systems in the era of globalization and information warfare, as well as determining the strategic reserve of stability for production cycles. The study object was mathematical methods of modelling production processes and determining the point of the production system stability. The study purpose was to use the fuzzy set device to determine the point of stability of the production system. To implement this study, methods of statistical analysis, data grouping, sample ranking, and methods for studying time series components were used. During the study, the scientific materials of leading researchers in the field of fuzzy sets and mathematical methods to calculate various components were used. The study results are intended for specialists and researchers in development and application of mathematical methods in the modelling of economic indicators of technological processes at enterprises.

Keywords: fuzzy set theory, strategic security, technical systems, stability domain.

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Актуальность применения теории нечётких множеств при расчёте стратегической безопасности сложной технической системы

Аннотация: Актуальность темы связана с проблемой защиты и безопасности технических систем в эпоху глобализации и информационной войны, а также определения стратегического запаса стабильности для производственных циклов. Объектом исследования стали математические методы моделирования производственных процессов и определение точки стабильности производственной системы. Целью исследования являлось применение аппарата нечётких множеств для определения точки стабильности производственной системы. Для реализации данного исследования были применены методы статистического анализа, группировка данных, ранжирование выборки, методы исследования компонент временного ряда. В ходе исследования были использованы научные материалы ведущих исследователей в области нечётких множеств и математических методов расчётах различных компонентов. Результаты исследования предназначаются для специалистов и исследователей в области разработки и применения математических методов в моделировании экономических показателей технологических процессов на предприятиях.

Abbreviation:
BEP is Break-even Point.

Introduction

The fuzzy sets concept was introduced by Lotfi Askar Zadeh back in 1965 in the article *Fuzzy Sets* in the journal *Information and Control* (*Zadeh, 1965*). He extended the classical set concept and allowed that the characteristic function of a set, or the membership function for a fuzzy set, can take any values in the interval specified. Set theory is the basic branch of discrete mathematics. The understanding of its other sections (graph theory, network theory, automata theory, etc.) is based on the set-theoretic representation of the processes of modelling objects of the surrounding world (*Zadeh et al., 1975*).

Fuzzy sets are based on solving the problem of processing and using fuzzy knowledge in image recognition systems. Fuzzy logic is one of the core technologies for developing efficient and efficient image recognition systems (*Volokhin, 2020*).

The relevance of the topic is related to the problem of protection and security of technical systems in the era of globalization and information warfare, as well as determining the strategic reserve of stability for production cycles. The variability of the area of stable economic activity of the enterprise makes it possible to flexibly plan all systems and modules. Therefore, it is necessary to develop mathematical models and methods for determining the strategic stability reserve for any production system.

The object of the study is the mathematical methods of modelling production processes. The subject of the study is to determine the point of stability of the production system.

The study purpose was to use the fuzzy set device to determine the point of stability of the production system.

Based on the goal, the following tasks were set during the study:

– develop a mathematical model for calculating the strategic stability reserve for the production system;

– develop a method for determining the strategic stability reserve for the production system;

– test the mathematical model of calculation and the method of determining the strategic stability reserve;

– determine whether it is possible to calculate the point of economic stability in parameters' range specified.

To implement this study, mathematical and statistical analysis, data grouping, sample ranking, and methods for studying time series components are used.

The research used scientific materials of L. A. Zadeh, G. Siegfried, R. E. Bellman, E. Polak, and other leading researchers in fuzzy sets and mathematical methods for calculating various components.

The study results are targeted at specialists and researchers in developing and applicating mathematical methods in the modelling of economic indicators of technological processes at enterprises.

Methods and Materials

One of the most important indicators of this process is the BEP Analysis. It allows determining the volume of production, sales, at which all costs associated with the manufacturing, are compensated. This indicator also forms the minimum required production capacity for successful operation.

By defining this point, we identify the volume, at which production breaks even when it is in it. However, if there is a deviation during economic activity influencing internal or external factors, the company 'goes into negative territory' and becomes unprofitable.

If there is a break-even point, then the existence of a stability points and a stability interval is logical. Having calculated it, the specialist receives a certain range of acceptable indicators, which is a criterion for maintaining economic stability. It helps to form the necessary counteraction and not reduce the efficiency of production. As a result, it is obvious that the indicators of conducting financial and economic (manufacturing) activities of the enterprise should be in the range of values of this desired interval.

Turning to the basic concepts of fuzzy set theory and fuzzy logic, it should note that the fuzzy system was the first proposed by the Polish mathematician Lukasiewicz in 1920. In fuzzy systems, the concept of weighted membership is introduced, i.e. an element can be more owned or less owned. In the 1960s, Lotfi Askar Zadeh, an American scientist of Azerbaijani origin, continued this work at the University of Berkeley.

Developing the *Theory of Fuzzy Sets*, Lotfi Askar Zadeh took steps to combine the accuracy of classical mathematics and the inaccuracy of the real world. The scientist believed that an excessive desire for accuracy began to have an effect that negates control theory and systems theory, leading to the fact that research in this field focuses exclusively on those problems that can be accurately solved. Many classes of significant problems in which the data, goals, and constraints are too complex or poorly defined to allow for accurate mathematical analysis have been left out for the sole reason that they are not amenable to mathematical interpretation.

Lotfi Askar Zadeh believed that the more complex a system is, the less the researcher can make accurate and at the same time practical judgments about its behaviour. In systems whose falsity exceeds the established threshold, accuracy and meaning become opposing parameters.

During the study, the advantages of fuzzy systems were identified:

- operating with qualitative and quantitative data;
- stability under the action of various perturbations on the system;
- building models of approximate human reasoning;
- operating under uncertainty;
- use of expert knowledge.

The advantages of fuzzy systems listed above were compared with the disadvantages of the system:

- no improvement in the accuracy of calculations compared to the probabilistic approach;

- impossibility of mathematical analysis by existing methods;
- the lack of a standard methodology for designing fuzzy systems.

In the *Theory of Fuzzy Sets*, the main approaches are probabilistic and fuzzy. Thus, fuzzy sets are a collection of elements, concerning which it is impossible to say with complete certainty whether they belong to a given set or not.

If it should write this expression mathematically, it is got:

A fuzzy set C is a collection of ordered pairs composed of elements x of the universal set X and the corresponding degrees of membership $\mu_C (X)$, which is a membership function that shows to what extent the element x belongs to the fuzzy set C.

$$C = \{(x, \mu_C (x)) | x \in X\}$$

Where:

the set of values of an x is tuple,

x is an element of the universe,

X is the universe itself.

Solving a Practical Problem

The results of the theoretical part of the research were verified based on a virtual production enterprise operating at the level of standard capacities and under conditions of certain dynamic stability. In the practical part of the study, an event was modelled that leads the enterprise out of the state of system stability.

It was proposed to introduce the concept of a stability domain with a set of stability points. It is obvious to assume that by representing the stability point as the value of the break-even point ((·) BEP) to which a 100% margin of reliability is added, it is expected to get a necessary and appropriate result. The best example to illustrate the correctness of this decision is the human body. For example, to open a door, one hand is needed, but to do this reliably, even under the influence of certain factors, the second hand provides absolute stability to this process.

The above can be represented as a formula:

$$(\cdot)STABILITY = (\cdot)BEP + 100\%$$

In this version, a question appears. Given that the problem is solved with respect to strategic enterprises and industries, which, due to their complexity, high cost and uniqueness, exist in the singular, then 100% in absolute value can represent hundreds of millions or billions of units of monetary equivalent. There is a strategic need to optimize or minimize the 'insurance' resources.

In this case, it is proposed to apply the apparatus of fuzzy sets to the problem being solved:

The values were phasified $(\cdot)STABILITY$:

0% is instability,

$y1$ is 25% is uncertain stability,

$y2$ is 50% is satisfactory stability,

$y3$ is 75% is stable stability,

$y4$ is 100% is full stability.

It is reasonable to assume that the phasification of the above values can remain constant, but change, if necessary, to use the "working terminology."

Next, the factors or influences affecting the system were phasified:

$x1$ is loss of customers

$x2$ is loss of quality

$x3$ is reduced customer demand

$x4$ is competitive activity

In this case, the above factors can be formed exclusively by expert means and will relate to a specific object of the application.

Thus, universum 1 is the factors or influences that affect the system:

$$X = \{x1, x2, x3, x4\}$$

Universum 2 has values of $(\cdot)STABILITY$:

$$Y = \{y1, y2, y3, y4\}$$

The mutual influence of universes can be expressed in tabular form, by expert evaluation of the interdependence values. These values can be either constant or refer to any set of values. Moreover, the values of the universum 2 can change over time, i.e., they can be dynamic characteristics that can be tracked using, e.g., a simulation model. The record will be made using the table 1 (*Appendix*).

The mathematical model of the function will have the form of a matrix:

$$Mf\,(\text{модель функция}) = \begin{vmatrix} 1 & 0{,}8 & 0{,}5 & 0{,}1 \\ 0{,}8 & 0{,}7 & 0{,}5 & 0{,}3 \\ 0{,}7 & 0{,}5 & 0{,}2 & 0{,}1 \\ 0{,}5 & 0{,}4 & 0{,}3 & 0{,}2 \end{vmatrix}$$

or a linear record:

F={(<X1,Y1>,1),(<X1,Y2>,0,8),(<X1,Y3>,0,5),(<X1,Y4>,0,1),
(<X2,Y1>,0,8),(<X2,Y2>,0,7),(<X2,Y3>,0,5),(<X2,Y4>,0,3),
(<X3,Y1>,0,7),(<X3,Y2>,0,5),(<X3,Y3>,0,2),(<X3,Y4>,0,1),
(<X4,Y1>,0,5),(<X4,Y2>,0,4),(<X4,Y3>,0,3),(<X4,Y4>,0,2) }

Fuzzy relationships should also be displayed as a graph (*Figure 1*):

Results

This article presents the study results of the possibility of using the fuzzy sets apparatus for solving such problems. It should consider that in the real production and industrial sector, it is necessary to conduct research using observations of dynamic characteristics. It is reasonable to assume that the practical implementation will require the creation of serious software support for the calculation, calculation and processing of the studied characteristics. In this study, using the minimax composition apparatus, the final solution of this function was made for one series of mutually influencing universes (*Figure 2*).

The values of minmax 0.3 and maxmin 0.3 are the same. Therefore, they are optimal.

Thus, the 30% value $(\cdot)STABILITY$: is optimal under the given conditions.

Discussion

The calculations carried out in the framework of the study gave a positive result. By representing the stability point as the break-even point $((\cdot)BEP)$ value, to which a 100% margin

of reliability is added, a result that uniquely satisfies all the system's security indicators was expected. However, the Theory of Fuzzy Sets application showed that in the situation modelled during the study, a 30-per cent margin is adequate in terms of reliability and optimality. The savings of 70% of the resources from the "insurance" reserve become obvious. Calculations showed that the saved resources are enough to ensure two more similar projects.

Conclusion

The apparatus of fuzzy sets used in this work turned out to be practically applicable and convenient for use. The empirical data obtained carry a huge practical potential for applicability. A significant feature of the proposed method is the possibility of saving more than half of the "insurance" resources. In the meantime, the security provided by a complex technical system is close to 1 or, in more understandable terms, the system is resistant to the expected possible threats.

Thus, as one of the study results, a conclusion was formed about the need to apply the theory of fuzzy sets to a wide range of practically solvable problems. Various expert values of universes were used in the calculations. Of course, they have a margin of error. However, even in this vein, there is an undeniable interest in the subsequent study of this issue. The author sees the main directions of further study of the development of methods for ensuring the security of a complex technical system in focusing on the development of methods using a computer, possibly with a dynamic component.

References:

Aleskovsky, V. V., & Yantsen A. V. (2011). Russian experience in developing a program-but-target approach. *Almanac of Modern Science and Education*, *4*(47), 156–158. (In Russ.)

Bellman, R. E., & Zadeh, L. A. (1970). Decision making in a fuzzy environment. *Management Science*, *17*(4), 141–164.

Bellman, R. E. & Zadeh, L. A. (1977). Local and fuzzy logics. *Modern Uses of Multiple-Valued Logic*, 103–165. The Netherlands: D. Reidel Publishing Co.

Chernov, V. P. (2002). *Mathematics for senior managers*. St. Petersburg: Nauka. (In Russ.)

Golik, E. S. (Ed.) (2005). *Mathematical methods of system analysis and decision-making theory. Part II: Textbook*. St. Petersburg: SZTU. (In Russ.)

Guts, A. K. (2016). *Mathematical logic and the theory of algorithms. Textbook*. Ed. 3rd, ISPR. Moscow: LENAND, 2016. (In Russ.)

International collection of Newspapers of the 19th and 20th century. (n.d.). (In Russ.). https://freep.newspapers.com/

Kim, E. S. (2016). Analysis of existing software to automate the operation of the enterprise. *Technology. Technologies. Engineering, 1*, 11–14.

Kolesnichenko, S. V. (2017). Information and statistical analysis of the processes of functioning of complex technical and socio-economic systems. *Proceedings of the 16th MNPK Analysis and Forecasting of Control Systems in Industry and Transport*, 336–348. St. Petersburg.: Fedorov Publishing House. (In Russ.)

Kolmogorov, A. N., & Dragalin, A. G. (2017). *Mathematical logic: Introduction to mathematical logic. Textbook*. Moscow: LENAND. (In Russ.)

Komissarov, P. V. (2018). Actual issues of modelling production processes of complex technical and socio-economic systems. *Collection of Works of the GUMRF*. St. Petersburg: GUMRF, 67–71. (In Russ.)

Komissarov, P. V. (2018). Method of complex assessment of economic indicators of the enterprise. *Collection: Analysis and Forecasting of Management Systems in Industry and Transport*, 57–58. St. Petersburg: Astra. (In Russ.)

Krass, M. S., & Chuprykov, B. P. (2019). *Mathematics in Economics: mathematical methods and models*. Moscow: Yurayt Publishing House. (In Russ.)

Leonenkov, A. V. (2003). *Fuzzy modeling in MATLAB and fuzzy TECH*. St Petersburg: BHV-Petersburg. (In Russ.)

Lisitsina, L. S. (2020). Fundamentals of fuzzy set theory. St Petersburg: University ITMO. (In Russ.)

Pyanykh, S. M. (1988). *Economic and mathematical methods of optimal planning of river transport*. Moscow: Transport. (In Russ.)

Siegfried, G. (2010). An early approach toward graded identity and graded membership in set theory. *Fuzzy Sets and Systems*, *161*(18), 2369–2379.

Volokhin, D. V. (2020). The relevance of the representation of fuzzy knowledge using fuzzy sets. *Scientific Aspect*, *2*(16), 2026–2029. Samara: Aspect. (In Russ.)

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353.

Zadeh, L. A., Fu, K., Tanaka, K., & Shimura M. (Eds.). (1975). Fuzzy sets and their applications to cognitive and decision processes. New York: Academic Press.

Zadeh, L. A., & Polak, E. (Eds.). (1969). System Theory. *Inter-University Electronics Series*, *8*. New York: McGraw-Hill.

Zemskov, A. V., & Shilkina I. D. (2018). *Economic and mathematical models. Educational and methodological guide for laboratory work*. St. Petersburg: Publishing house of S. O. Makarov GUMRF. (In Russ.)

Appendix

Table 1. Indexing universum 1 factors

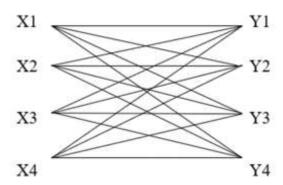|	y1	y2	y3	y4
x1	1	0,8	0,5	0,1
x2	0,8	0,7	0,5	0,3
x3	0,7	0,5	0,2	0,1
x4	0,5	0,4	0,3	0,2

Figure 1. The graph of fuzzy relationships

Figure 2. Using the minimax composition apparatus, the final solution of the function for one series of mutually influencing universes